Exercise 3.3.43 Consider $f(x) = x^2 + 2x$.
(a) Find the vertex, axis of symmetry, and determine whether the graph is concave up or concave down. (b) Find the y-intercept and x-intercepts, if any. (c) Use parts (a) and (b) to graph. (d) Find the domain and range. (e) Determine where the quadratic function is increasing and where it is decreasing. (f) Determine where $f(x) > 0$ and where $f(x) < 0$.

Solution

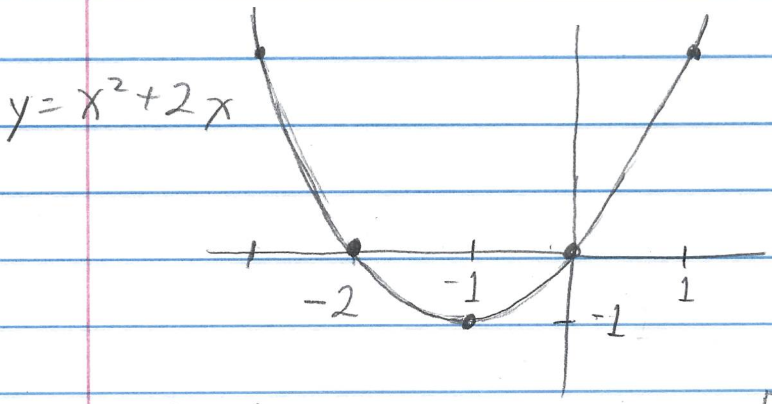
(a) For $f(x) = x^2 + 2x$ we have $a = 1$, $b = 2$, and $c = 0$. By Note 3.3.A, the vertex is $(-b/(2a), f(-b/(2a)))$ and the axis of symmetry is $x = -b/(2a)$. Since $-b/(2a) = -(2)/(2(1)) = -1$, then

$$\boxed{\begin{array}{l}\text{the vertex is } (-1, (-1)^2 + 2(-1)) = (-1, -1) \\ \text{and the axis of symmetry is } x = -1. \\ \text{Since } a = 1 > 0 \text{ then the graph is concave up.}\end{array}}$$

(b) For the y-intercept, we set $x = 0$ to get $f(0) = (0)^2 + 2(0) = 0$, so $\boxed{\text{the y-intercept is } 0}$. For the x-intercepts, we set $y = f(x) = 0$ and consider $f(x) = x^2 + 2x = x(x+2) = 0$. So

$\boxed{\text{the x-intercepts are } 0 \text{ and } -2.}$

(c) Based on the intercepts, the vertex, and the fact that the graph is a parabola we have:



(We also notice that $f(-3) = f(1) = 3$.)

(d) Based on the graph, the domain of f is all real numbers, $\mathbb{R} = (-\infty, \infty)$, and the range is $[-1, \infty)$.

(e) Based on the graph, the function is decreasing for $x \in (-\infty, -1)$ (where the function is going "downhill" when read from left to right) and the function is increasing for $x \in (-1, \infty)$ (where the function is going "uphill" when read from left to right).

(f) Based on the graph, $f(x) > 0$ for $x \in (-\infty, -2) \cup (0, \infty)$, and $f(x) < 0$ for $x \in (-2, 0)$.

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