

Exercise 3.3.71 Consider $f(x) = -5x^2 + 20x + 3$.

Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value, and then find the value.

Solution

For $f(x) = -5x^2 + 20x + 3$, we have $a = -5$, $b = 20$, and $c = 3$. Since $a = -5 < 0$ then the graph of $y = f(x)$ is concave down.

Since the graph of this quadratic function is a parabola, then there is a highest point on the graph and so f has a maximum.

The maximum will occur at the vertex $(-b/(2a), f(-b/(2a)))$ (by Note 3.3.A).

Since $-b/(2a) = -(20)/(2(-5)) = 2$ and $f(2) = -5(2)^2 + 20(2) + 3 = 23$, then

the maximum of f is 23. \square