

Exercise 3.4.13 A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90° .

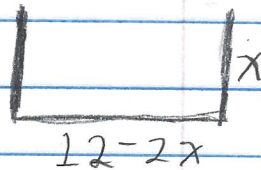
(a) What depth will provide maximum cross-sectional area and allow the most water to flow?

(b) What depths will allow at least 16 square inches of water to flow?

Solution

Based on the figure in the book, the cross-section has dimensions:

So the cross-sectional area is $A(x) = (12 - 2x)x$.



(a) With $A(x) = 12x - 2x^2$, in the form $(ax^2 + bx + c)$ we have $a = -2$, $b = 12$, and $c = 0$.

Since $a = -2 < 0$, the graph of $A(x)$ is concave down and so the maximum of $A(x)$ occurs at the vertex $(-b/(2a), A(-b/(2a)))$.

So the depth x that maximizes cross-sectional area is $x = -b/(2a) = -(12)/(2(-2)) = \boxed{3 \text{ inches}}$.

(b) With $A(x) = 12x - 2x^2 = 16$ we need $-2x^2 + 12x - 16 = 0$ or $-x^2 + 6x - 8 = 0$ or $(-x + 2)(x - 4) = 0$. We see that $A(2) = A(4) = 16$.

Since A is concave down then from the shape of the graph we have $A(x) \geq 16$ for $2 \leq x \leq 4$.

So the depth should be in the interval $[2, 4]$.

□