Exercise 3.4.5 The price p (in dollars) and the quantity x sold of a certain product satisfy the demand equation $x = -5p + 100$.

(a) Find a model that expresses the revenue R as a function of p.

(b) What is the domain of R? Assume $R \geq 0$.

(c) What price p maximizes the revenue?

(d) What is the maximum revenue?

(e) How many units are sold at this price?

(f) Graph R.

(g) What price should the company charge to earn at least \$480 in revenue?

Solution

(a) At a price of p dollars each, with x units sold the revenue function is $R = px$. Since $x = -5p + 100$ then $R(p) \equiv p(-5p + 100)$ or $\boxed{R(p) = -5p^2 + 100p \text{ dollars.}}$

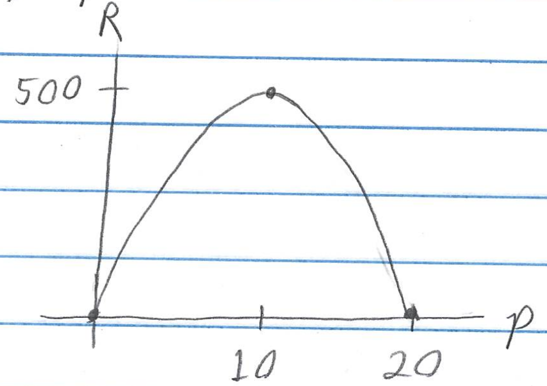
(b) Since p is a price, then $p \geq 0$. Since $R(p) = p(-5p + 100) \geq 0$ then we need $-5p + 100 \geq 0$ or $5p \leq 100$ or $p \leq 20$. So the domain of R is $\boxed{p \in [0, 20]}$.

(c) Since $R(p) = -5p^2 + 100p$, then in the form $ap^2 + bp + c$ we have $a = -5$, $b = 100$, and $c = 0$. Since $a = -5 < 0$ then the graph of $R(p)$ is concave down and so the maximum of $R(p)$ occurs at the vertex $(-b/(2a), R(-b/(2a)))$. So the price that maximizes revenue is $p = -b/(2a)$ $= -(100)/(2(-5)) = \boxed{10 \text{ dollars.}}$

(d) The maximum revenue is $R(-b/(2a)) = R(10) = -5(10)^2 + 100(10)$ $= -500 + 1000 = \boxed{500 \text{ dollars.}}$

(e) At the price $p = \$10$ that maximizes revenue, the quantity sold (or "demanded") is $x = -5p + 100 = -5(10) + 100 = \boxed{50 \text{ units}}$.

(f) Since $R(p) = p(-5p + 100)$, then the intercepts are $p = 0$ and $p = 20$ where $R(p) = 0$. The graph is then



(g) With $R(p) = -5p^2 + 100p \doteq 480$ we need $-5p^2 + 100p - 480 \doteq 0$ or $-p^2 + 20p - 96 \doteq 0$ or $(-p + 8)(p - 12) \doteq 0$. We see that $R(8) = R(12) = \$480$. Since R is concave down then we see from the shape of the graph that $R(p) \geq 480$ for $8 \leq p \leq 12$. So the $\boxed{\text{price should be in the interval } [8,12].}$

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