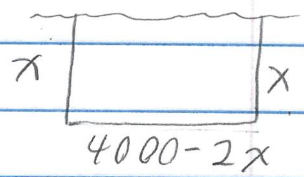
Exercise 3.4.9 A farmer with 4000 meters of fencing wants to enclose a rectangular plot that borders a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?

Solution

We are given the figure:



The area of a rectangle is base × height, so the area as a function of x is $A(x) = (4000-2x)x = 4000x - 2x^2$. Then in the form ax^2+bx+c we have $a=-2$, $b=4000$, and $c=0$. Since $a=-2<0$ then the graph of $A(x)$ is concave down and so the maximum of $A(x)$ occurs at the vertex $(-b/(2a), A(-b/(2a)))$. So area is maximized when $x = -b/(2a) = -(4000)/(2(-2))$ $= 1000$ meters and the maximum is

$$A(1000) = 4000(1000) - 2(1000)^2$$
$$= 4{,}000{,}000 - 2{,}000{,}000$$
$$= \boxed{2{,}000{,}000 \text{ meters}^2}.$$

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