Exercise 3.5.17 Solve the inequality $x^2 - x + 1 \leq 0$.

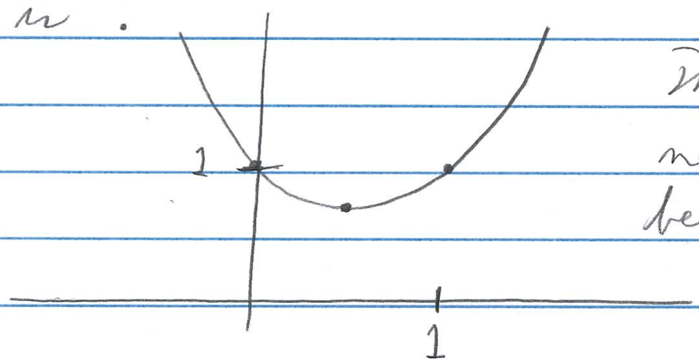
Solution

Let $f(x) = x^2 - x + 1$ and we consider $f(x) \leq 0$. With $f(x)$ in the form ax^2+bx+c we have $a=1$, $b=-1$, and $c=1$. Since $a = 1 > 0$ is concave up. So we have $f(x) \leq 0$ between the x-intercepts of the graph of $y = f(x)$. To find the x-intercepts, we set $f(x) = x^2 - x + 1 = 0$. To apply the quadratic formula, we first consider the discriminant $b^2 - 4ac = (-1)^2 - 4(1)(1) = -3 < 0$. But with a negative discriminant, there are no x-intercepts and hence there are $\boxed{\text{no real numbers } x \text{ with } x^2 - x + 1 \leq 0.}$ □

Note The vertex of $y = x^2 - x + 1$ is at

$$(-b/(2a), f(-b/(2a))) = (-(-1)/(2(1)), f(-(-1)/(2(1)))$$

$$= \left(\frac{1}{2}, \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1\right) = \left(\frac{1}{2}, \frac{3}{4}\right),$$ so the graph is:



The graph is never on or below the x-axis.