Exercise 3.5.29 Consider $f(x) = x^2 - x - 2$ and $g(x) = x^2 + x - 2$. (a) Solve $f(x) = 0$. (b) Solve $g(x) = 0$. (c) Solve $f(x) = g(x)$. (d) Solve $f(x) > 0$. (e) Solve $g(x) \le 0$. (f) Solve $f(x) > g(x)$. (g) Solve $f(x) \ge 1$.

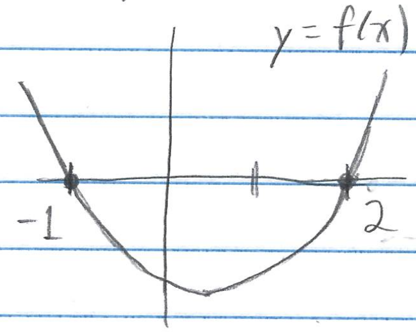
Solution

(a) We consider $f(x) = x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$. So either $\boxed{x = -1 \text{ or } x = 2.}$

(b) We consider $g(x) = x^2 + x - 2 = 0$ or $(x+2)(x-1) = 0$. So either $\boxed{x = -2 \text{ or } x = 1.}$

(c) We consider $f(x) = x^2 - x - 2 = x^2 + x - 2 = g(x)$ or $-x = x$ or $2x = 0$. So $\boxed{x = 0.}$

(d) We consider $f(x) = x^2 - x - 2 > 0$. Since $a = 1 > 0$ then the graph of $y = f(x)$ is concave up and so the inequality is satisfied "outside" of the x-intercepts:

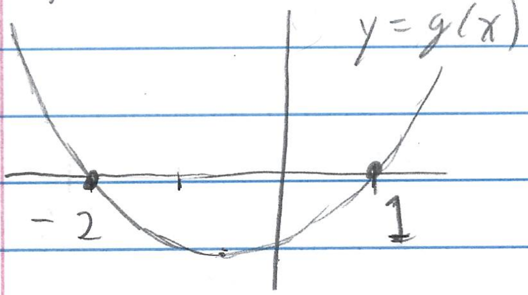


Since the x-intercepts are $x = -1$ and $x = 2$ by (a), then the $\boxed{\text{inequality is satisfied for } x \in (-\infty, -1) \cup (2, \infty).}$

(e) We consider $g(x) = x^2 + x - 2 \le 0$. Since $a = 1 > 0$ then the graph of $y = g(x)$ is concave up and so the inequality is satisfied between the x-intercepts

3.5.29 continued

(and including the x-intercepts, for the equality):



Since the x-intercepts are $x = -2$ and $x = 1$ by (b), then the $\boxed{\text{inequality is satisfied for } x \in [-2, 1]}$.

(f) We consider $f(x) = x^2 - x - 2 > x^2 + x - 2 = g(x)$ or $-x > x$ or $0 > 2x$ or $0 > x$ or $x < 0$. So $\boxed{\text{the inequality is satisfied for } x \in (-\infty, 0).}$

(g) We consider $f(x) = x^2 - x - 2 \geq 1$ or $x^2 - x - 3 \geq 0$. Define $h(x) = x^2 - x - 3$. Since $a = 1 > 0$ then the graph of $y = h(x)$ is concave up and so the inequality is satisfied "outside" of the x-intercepts (as in (d)), and including the intercepts. We use the quadratic formula to find the x-intercepts and consider

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \frac{1}{2} \pm \frac{\sqrt{13}}{2},$$

or $x = \frac{1}{2} - \frac{\sqrt{13}}{2}$ and $x = \frac{1}{2} + \frac{\sqrt{13}}{2}$.

So the inequality is satisfied for

$$\boxed{x \in \left(-\infty, \frac{1}{2} - \frac{\sqrt{13}}{2}\right] \cup \left[\frac{1}{2} + \frac{\sqrt{13}}{2}, \infty\right).}$$

$\square$