(4.1.49)

Exercise 4.1.49 Find a polynomial of degree $n=3$ with the real zeros $r_1=-2$, $r_2=3$, and $r_3=5$ whose graph contains the point $(2,36)$.

Solution

Since $r_1=-2$ is a zero of the polynomial p then $(x-r_1)^{m_1}=(x-(-2))^{m_1}=(x+2)^{m_1}$ is a factor of p for some integer $m_1 \geq 1$. Similarly, $(x-r_2)^{m_2}=(x-3)^{m_2}$ is a factor of p for some integer $m_2 \geq 1$, and $(x-r_3)^{m_3}=(x-5)^{m_3}$ is a factor of p for some integer $m_3 \geq 1$. Therefore $(x+2)^{m_1}(x-3)^{m_2}(x-5)^{m_3}$ is a factor of p. Since this factor is of degree $m_1+m_2+m_3$ where $3 \leq m_1+m_2+m_3$ and p is of degree 3, then we must have $m_1=m_2=m_3$ and there are no other polynomial factors of p of positive degree. So p must be of the form $p(x)=a_3(x+2)(x-3)(x-5)$. Since $(2,36)$ is a point on the graph of p then $p(2)=36$ and so

$p(2)=a_3((2)+2)((2)-3)((2)-5)=36$

or $12a_3=36$ and hence $a_3=36/12=3$.

Therefore $\boxed{p(x)=3(x+2)(x-3)(x-5).}$ □