Exercise 4.1.61 Consider $f(x) = 7(x^2+4)^2(x-5)^3$.
(a) List each real zero and its multiplicity.
(b) Determine whether the graph crosses or touches the x-axis at each x-intercept.
(c) Determine the maximum number of turning points on the graph.
(d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

Solution

(a) Since x^2+4 is irreducible and has no real zeros ($x^2 = -4$ has no real solutions), then $\boxed{\text{the only real zero is } x=5, \text{ of multiplicity } 3.}$

(b) By (a), the only x-intercept is $x=5$. Since this is a zero of multiplicity 3 (odd), then the graph $\boxed{\text{crosses the x-axis at } x=5.}$

(c) If we multiply out $f(x)$, we see that the degree is $n = 2\times 2+3 = 7$. So by Theorem 4.1.A $\boxed{\text{the maximum number of turning points is } n-1=6.}$

(d) By Theorem 4.1.B, the end behavior of $f(x) = 7(x^2+4)^2(x-5)^3 = 7(x^4+8x^2+16)(x^3+3(x^2)(-5) + 3(x)(-5)^2 - 125)$ is $\boxed{a_n x^n = 7x^7}$, as we see by multiplying out and considering the first expression for f in standard form. □