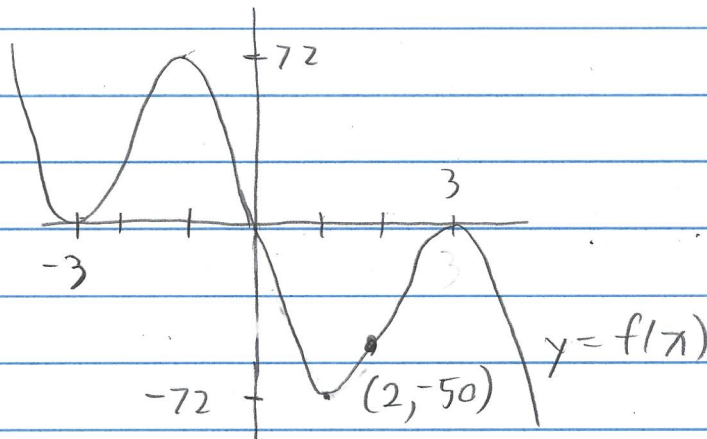


Exercise 4.1.81 Write a polynomial function whose graph is (explain your reasoning):



Solution

Since the x -intercepts are $x = -3$, $x = 0$, $x = 3$ then $(x - (-3)) = (x + 3)$, $x - 0 = x$, and $(x - 3)$ are factors of f . Since the graph touches the x -axis at $x = -3$ and $x = 3$ then the factors $(x + 3)$ and $(x - 3)$ are of even multiplicity. Since the graph crosses the x -axis at $x = 0$ then the factor x must be of odd multiplicity. Since the graph has 4 turning points, then f must be of degree at least 5 by Theorem 4.1.4. We try $f(x) = a_5 x(x - 3)^2(x + 3)^2$. Since $(2, -50)$ is on the graph of f then $f(2) = a_5(2)(2 - 3)^2(2 + 3)^2 = -50$ or $50a_5 = -50$ or $a_5 = -1$. So we take $f(x) = -x(x - 3)^2(x + 3)^2$.

□