4.2.17

Exercise 4.2.17 Graph $f(x) = -2(x-1)^2(x^2-16)$ by following Steps 1 through 5.

Solution

Step 1 Determine the end behavior. We have $f(x) = -2(x-1)^2(x^2-16)$ so that f is a degree $n=4$ polynomial function with leading term (when multiplied out) of $-x^4$, so the end behavior is $\boxed{y=-x^4}$.

Step 2 Find the x and y intercepts.

For the y-intercept, we set $x=0$ and get $f(0) = -2((0)-1)^2((0)^2-16) = (-2)(-1)^2(-16) = 32$. So the $\boxed{y\text{-intercept is } 32.}$ For the x-intercept we set $y = f(x) = 0$ and consider $-2(x-1)^2(x^2-16) = 0$ or $-2(x-1)^2(x-4)(x+4) = 0$. So the $\boxed{x\text{-intercepts are } -4, 1, \text{ and } 4.}$

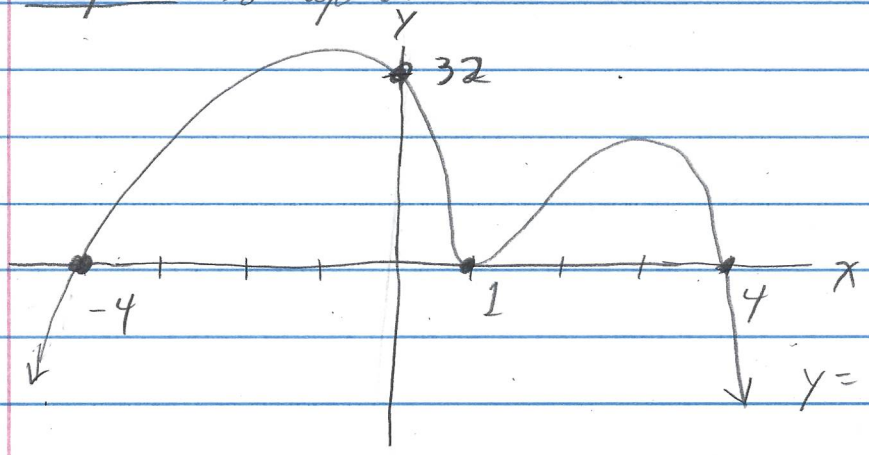
Step 3 Determine the multiplicities of the zeros and whether the graph crosses or touches the x-axis at each x-intercept. Since $f(x) = -2(x-1)^2(x-4)(x+4)$, then $\boxed{\text{zero } -4 \text{ is of multiplicity 1, zero 1 is of multiplicity 2, and zero 4 is of multiplicity 1.}}$ At a zero of even multiplicity the graph touches the x-axis and at a zero of odd multiplicity the graph crosses the x-axis (by Note 4.1.C).

$\boxed{\text{So the graph crosses the } x\text{-axis at } x=-4 \text{ and } x=4 \text{ and touches the } x\text{-axis at } x=1.}$

4.2.17 continued

Step 4 Determine the maximum number of turning points. Since f is a polynomial function of degree $n=4$, then by Theorem 4.1.A the maximum number of turning points is $n-1=3$.

Step 5 Graph. We have



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