Exercise 4.2.21 Graph $f(x) = x^2(x-2)(x^2+3)$ by following Steps 1 through 5.

Solution

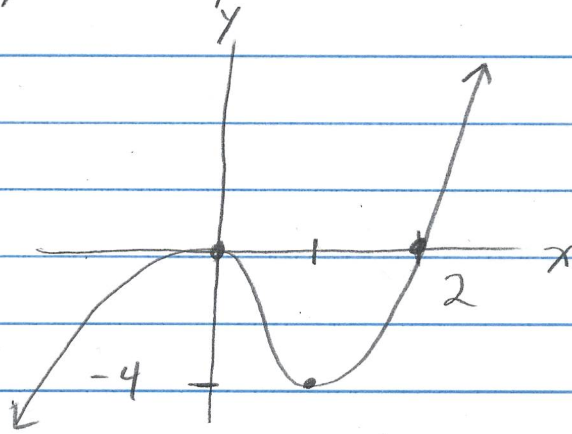
Step 1 Determine the end behavior. We have $f(x) = x^2(x-2)(x^2+3)$ so that f is a degree $n=5$ polynomial function with leading term (when multiplied out) of x^5. So the end behavior is $\boxed{y = x^5.}$

Step 2 Find the x and y-intercepts.
For the y-intercept, we set $x=0$ and get $f(0) = (0)^2((0)-2)((0)^2+3) = 0$. So the $\boxed{\text{y-intercept is } 0.}$ For the x-intercept we set $y = f(x) = x^2(x-2)(x^2+3) = 0$. Since $x^2+3=0$ has no solution, so the $\boxed{\text{x-intercepts are } 0 \text{ and } 2}$.

Step 3 Determine the multiplicities of the zeros and whether the graph crosses or touches the x-axis at each x-intercept. Since $f(x) = x^2(x-2)(x^2+3)$, then $\boxed{\text{zero } 0 \text{ is of multiplicity } 2 \text{ and zero } 2 \text{ is of multiplicity } 1.}$ At a zero of even multiplicity the graph touches the x-axis and at a zero of odd multiplicity the graph crosses the x-axis (by Note 4.1.C).
$\boxed{\text{So the graph crosses the x-axis at } x=2 \text{ and touches the x-axis at } x=0.}$

Step 4 Determine the maximum number of turning points. Since f is a polynomial function of degree $n=5$, then by Theorem 4.1.A the maximum number of turning points is $n-1=4$.

Step 5 Graph. Notice $f(1)=(1)^2((1)-2)((1)^2+3)=-4$.



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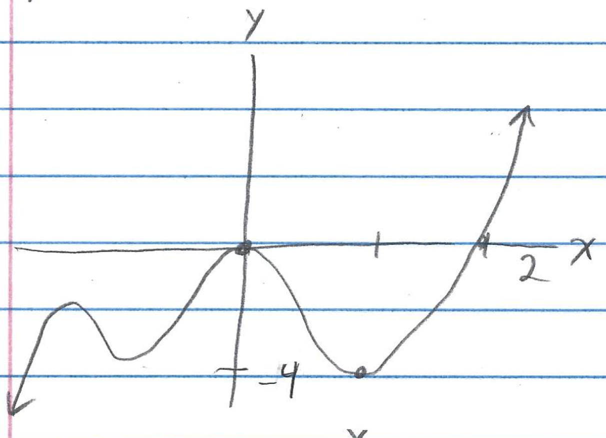
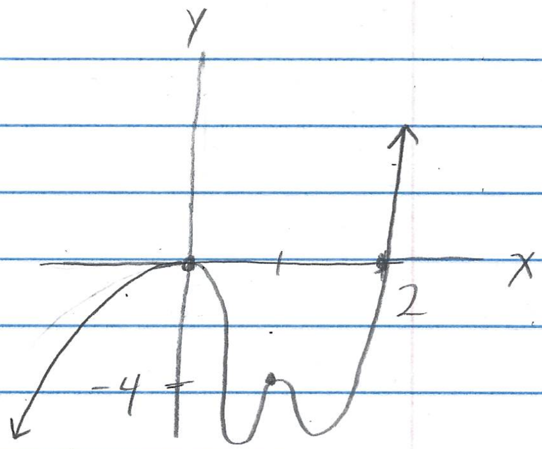
Note The graph **could** have 2 more turning points and could be:





or