(4.2.37)

Exercise 4.2.37 Graph $f(x) = -x^5 - x^4 + x^3 + x^2$ by following Steps 1 through 5.

Solution

Step 1 Determine the end behavior. We have $f(x) = -x^5 - x^4 + x^3 - x^2$ so the end behavior is the leading term $\boxed{y = -x^5}$.

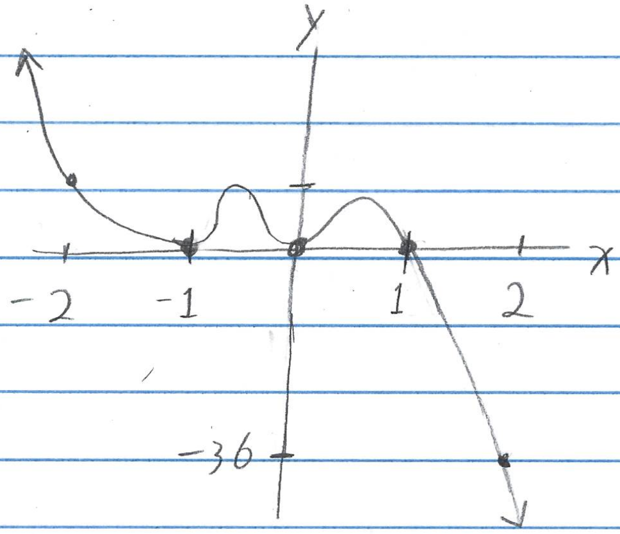
Step 2 Find the x and y-intercepts. For the y-intercepts, we set $x = 0$ and get $f(0) = -(0)^5 - (0)^4 + (0)^3 + (0)^2 = 0$. So $\boxed{\text{the } y\text{-intercept is } 0.}$ For the x-intercept we set $y = f(x) = 0$ and consider $-x^5 - x^4 + x^3 + x^2 = 0$ or $-x^2(x^3 + x^2 - x - 1) = 0$ or $-x^2(x^2(x+1) - (x+1)) = 0$ or $-x^2(x^2 - 1)(x+1) = 0$ or $-x^2(x-1)(x+1)(x+1) = 0$ or $-x^2(x-1)(x+1)^2 = 0$. So the $\boxed{x\text{-intercepts are } -1, 0, \text{ and } 1.}$

Step 3 Determine the multiplicities of the zeros and whether the graph crosses or touches the x-axis at each x-intercept. Since $f(x) = -x^2(x-1)(x+1)^2$ then $\boxed{\text{zero } -1 \text{ is of multiplicity } 2, \text{ zero } 0 \text{ is of multiplicity } 2, \text{ and zero } 1 \text{ is of multiplicity } 1.}$ At a zero of even multiplicity the graph touches the x-axis and at a zero of odd multiplicity the graph crosses the x-axis (by Note 4.1.C). So $\boxed{\text{the graph crosses the } x\text{-axis at } x = 1 \text{ and touches the } x\text{-axis at } x = -1 \text{ and } x = 0.}$

4.2.37 continued

Step 4 Determine the maximum number of turning points. Since f is a polynomial function of degree $n=5$, then by Theorem 4.1.A the maximum number of turning points is $n-1=4$.

Step 5 Graph. Notice $f(-2) = -(-2)^5 - (-2)^4 + (-2)^3 + (-2)^2 = 32 - 16 - 8 + 4 = 12$ and $f(2) = -(2)^5 - (2)^4 + (2)^3 + (2)^2 = -32 - 16 + 8 + 4 = -36$.



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