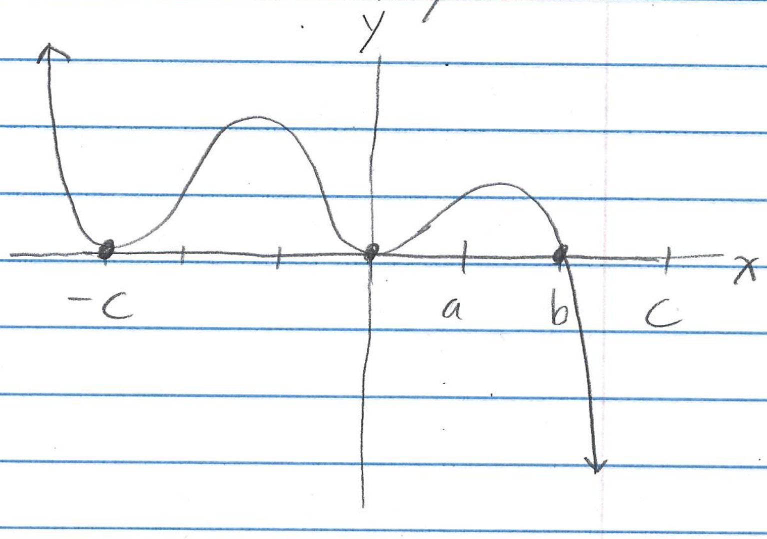
4.2.49

Exercise 4.2.49 Suppose $f(x) = -ax^2(x-b)(x+c)^2$, where $0 < a < b < c$.
(a) Graph f. (b) On what interval is there a local maximum value? (c) Which numbers yield a local minimum value?
(d) Where is $f(x) < 0$? Where is $f(-x-4) < 0$?
(f) Is f increasing, decreasing, or neither on $(-\infty, -c]$?

Solution

(a) Notice that the end behavior (Step 1) is $y = -ax^5$. The x-intercepts are $x = 0$ (of multiplicity 2), $x = b$ (of multiplicity 1), and $x = +c$ (of multiplicity 2). So the graph crosses the x-axis at $x = b$ and touches the x-axis at $x = +c$ and $x = 0$ by Theorem 4.1.C (Steps 2 and 3). Since the degree is $n = 5$ then there are at most $n - 1 = 4$ turning points. The end behavior $y = -ax^5$ tells us that the graph is positive for x large and negative and the graph is negative when x is large and positive.



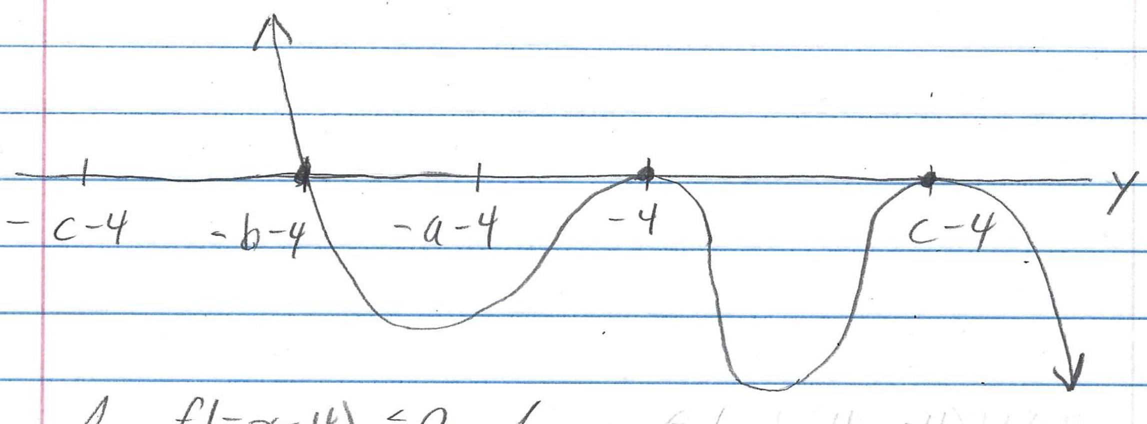
4.2.49 continued

(b) We see from the graph that there is a local maximum in the interval $(-c, 0)$ and a local maximum in the interval $(0, b)$.

(c) There is a local minimum of 0 at $x = -c$ and a local minimum of 0 at $x = 0$.

(d) From the graph, $f(x) < 0$ for $x \in (b, \infty)$.

To consider $f(-x-4)$, we take the graph of $y = f(x)$ and replace x with $-x$ (which corresponds to a refection about the y-axis) to get $y = f(-x)$. Next we replace x with $x + 4 = x - (-4)$ (which corresponds to a horizontal transformation left by 4 units) to get $y = f(-x+4)$ with graph:



So $f(-x-4) < 0$ for

$$\boxed{x \in (-b-4, -4) \cup (-4, c-4) \cup (c-4, 0).}$$

(f) From the graph of $y = f(x)$, we see that f is decreasing on $(-\infty, -c]$.

$\square$