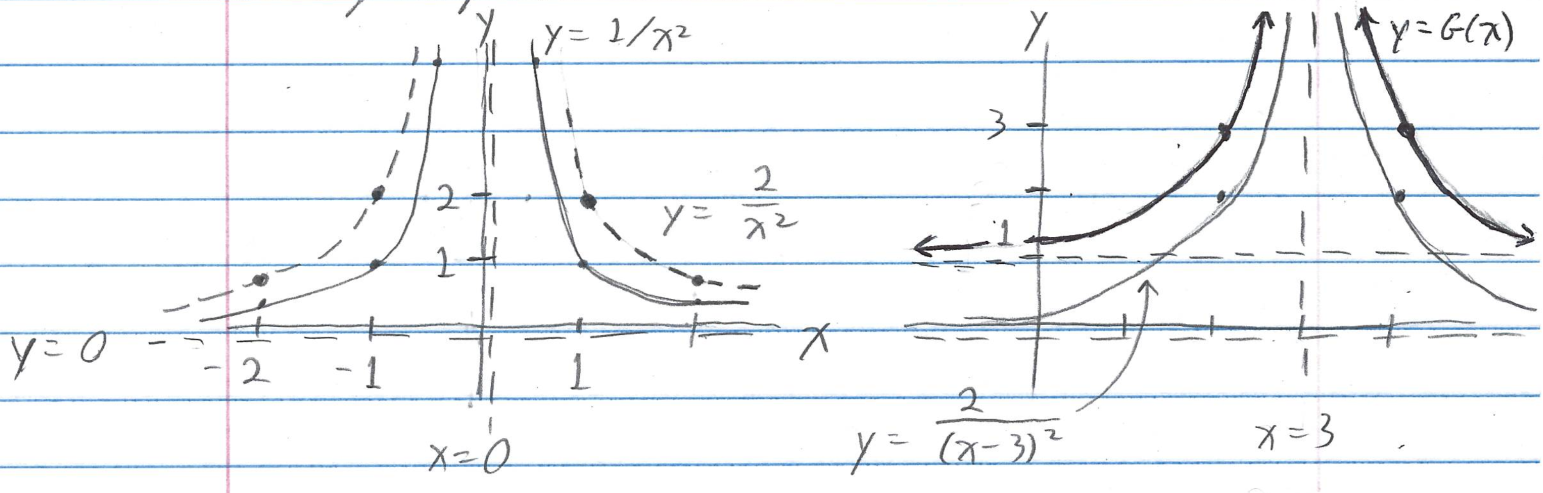
Exercise 4.3.41 Consider $G(x) = 1 + \frac{2}{(x-3)^2}$.
(a) Graph using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

Solution

(a) We start with the graph of $y = 1/x^2$ (now in the Library of Functions). We multiply $1/x^2$ by 2 (which corresponds to a vertical stretch by a factor of 2 to get $y=2/x^2$. Next replace $x$ with $x-3$ (which corresponds to a horizontal transformation to the right by 3 units) to get $y=2/(x-3)^2$. Finally add 1 (which corresponds to a vertical shift up by 1 unit) to get $y = G(x) = 1 + 2/(x-3)^2$. We have




(b) From the graph of $y = G(x)$ we see that the domain is $(-\infty, 3) \cup (3, \infty)$, and the range is $(1, \infty)$.

(c) We see from the graph that the $\boxed{\text{horizontal asymptote is } y = 1, \text{ the vertical asymptote is } x = 3, \text{ and there is no oblique asymptote.}}$ □

Note If we vertically stretch by a factor of 2, horizontally transform to the right by 3 units, and vertically transform up by 1 unit the horizontal asymptote $y = 0$ of $y = 1/x^2$, then we get the horizontal asymptote for $y = 1$ of $y = G(x)$. Similarly, the vertical asymptote $x = 0$ of $y = 1/x^2$ transforms to the vertical asymptote $x = 3$ of $y = G(x)$.