

Exercise 4.3.49 Consider $T(x) = \frac{x^3}{x^4-1}$.

Find the vertical, horizontal, and oblique asymptotes, if any, of $y = T(x)$.

Solution

Rational function T has a numerator of degree $n=3$ and a denominator of degree $m=4$. So by Note 4.2.B(1) (since

$3 = n < m = 4$), the line $y=0$ is a horizontal asymptote and (by Note 4.2.B(3)) there is no oblique asymptote.

$$\begin{aligned} \text{Notice that } T(x) &= \frac{x^3}{x^4-1} = \frac{x^3}{(x^2-1)(x^2+1)} \\ &= \frac{x^3}{(x-1)(x+1)(x^2+1)} \end{aligned}$$

in lowest terms. So by Theorem 2.2.A,

T has vertical asymptotes of $x=-1$ and $x=1$, since the denominator is 0 at $x=-1$ and $x=1$. Notice that x^2+1 is never 0. \square