

4.3.61

Exercise 4.3.61 The standard form of the rational function $R(x) = \frac{mx+b}{cx+d}$, $c \neq 0$,

is $R(x) = a\left(\frac{1}{x-h}\right) + k$. To write the rational function in standard form requires polynomial division.

(a) Write the rational function $R(x) = \frac{2x+3}{x-1}$ in standard form.

(b) Graph R using transformations.

(c) Find the vertical asymptote and the horizontal asymptote of R .

Solution

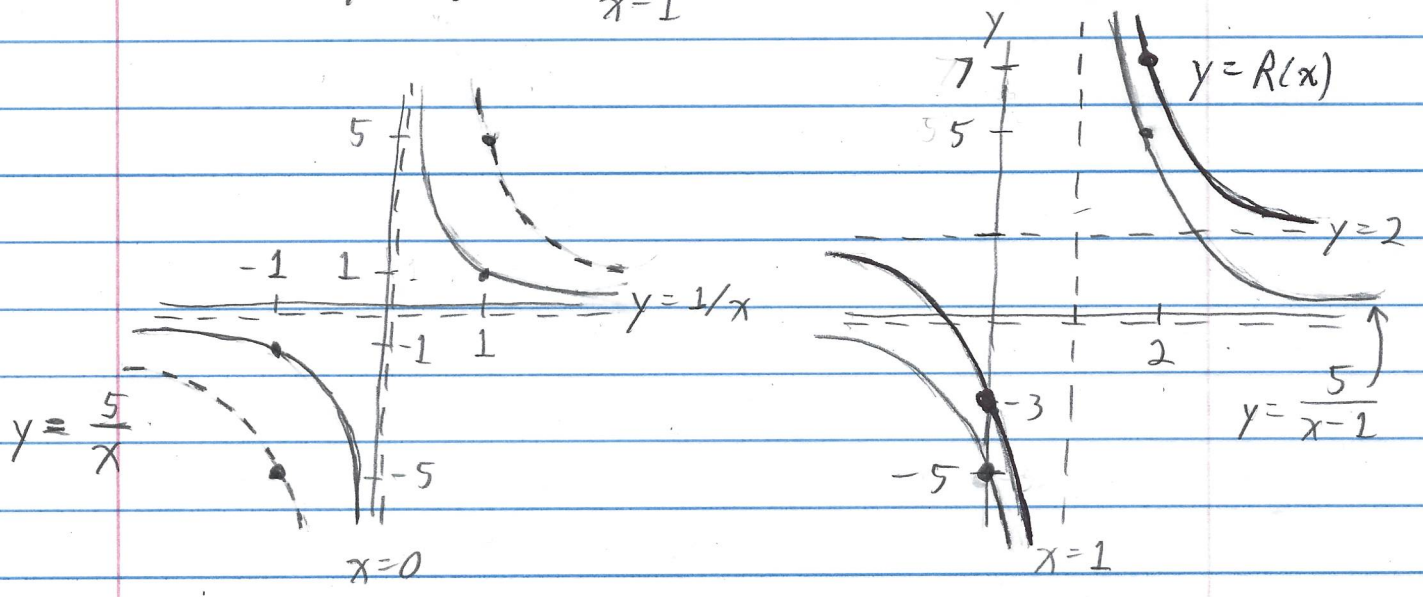
(a) First, we divide

$$\begin{array}{r} 2 \\ x-1 \overline{) 2x+3} \\ \underline{-(2x-2)} \\ 5 \end{array}$$

so $R(x) = \frac{2x+3}{x-1} = 2 + \frac{5}{x-1}$ in standard form.

(b) We start with the library of functions $y = 1/x$ and first multiply by 5 (which corresponds to a vertical stretch by a factor of 5) to get $y = 5/x$. Next, we replace x by $x-1$ (which corresponds to a horizontal shift to the right by 1 unit), to get $y = \frac{5}{x-1}$. Finally, we add 2

(which corresponds to a vertical shift by 2 units)
to get $y = 2 + \frac{5}{x-1} = R(x)$. We have:



(c) From the graph, we see that the vertical asymptote of $y = R(x)$ is $x = 1$ and the horizontal asymptote is $y = 2$. □

Note If we apply the three transformations to the asymptotes $x = 0$ and $y = 0$ of $y = 1/x$, then we get the asymptotes $x = 1$ and $y = 2$ of $y = R(x)$.