4.4.17

Exercise 4.4.17 Consider $R(x) = \frac{x^2}{x^2+x-6}$. Graph R by following Steps 1 through 7.

Solution

Step 1 Factor the numerator and denominator and find the domain. We have

$$\boxed{R(x) = \frac{x^2}{x^2+x-6} = \frac{x^2}{(x+3)(x-2)}}.$$

The domain excludes -3 and 2 so the $\boxed{\text{domain is } (-\infty,-3)\cup(-3,2)\cup(2,\infty).}$

Step 2 Put the rational function in lowest terms. Since the numerator and denominator have no common factors then R is in lowest terms as $\boxed{R(x) = \frac{x^2}{(x+3)(x-2)}}$.

Step 3 Locate the intercepts and describe the behavior at each x-intercept.

For the y-intercept set $x=0$ and we get

$$R(0) = \frac{(0)^2}{((0)+3)((0)-2)} = \frac{0}{-6} = 0,$$ so

$\boxed{\text{the y-intercept is } 0.}$ For x-intercepts we set $y = R(x) = 0$ and consider $x^2 = 0$.

$\boxed{\text{So the x-intercept is } 0.}$

Step 4 Find the vertical asymptotes. We consider R in lowest terms and see that the vertical asymptotes occur where the denominator $x^2+x-6 = (x+3)(x-2)$ is 0.

So the vertical asymptotes are $x = -3$ and $x = 2$.

Step 5 Locate the horizontal and oblique asymptotes and find points, if any, where the graph intersects them. Since the numerator and denominator are both of degree $n = m = 2$ then by Note 4.2.B(2), there is a horizontal asymptote of $y = a_n / b_m = (1)/(1) = 1$.

By Note 4.2.B(3), there is no oblique asymptote.

To find points where $y = R(x)$ intersects horizontal asymptote $y = 1$, we consider

$$R(x) = \frac{x^2}{x^2 + x - 6} = 1 \text{ or } x^2 = x^2 + x - 6$$

or $x - 6 = 0$ or $x = 6$. So the graph intersects the horizontal asymptote at $x = 6$.

Step 6 Determine where the graph is above or below the x-axis. We remove the zeros of the numerator and denominator from $\mathbb{R}$ and test the sign of R on these intervals. We get the intervals $(-\infty, -3)$, $(-3, 0)$, $(0, 2)$, $(2, \infty)$ and consider:

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 2)$	$(2, \infty)$
Test Value c	-4	-1	1	3
Value of $R(c)$	$16/6$	$1/(-6)$	$1/(-4)$	$9/6$
Conclusion	R positive	R negative	R negative	R positive

So R is above the x-axis on $(-\infty, -3) \cup (2, \infty)$ and R is below the x-axis on $(-3, 0) \cup (0, 2)$.

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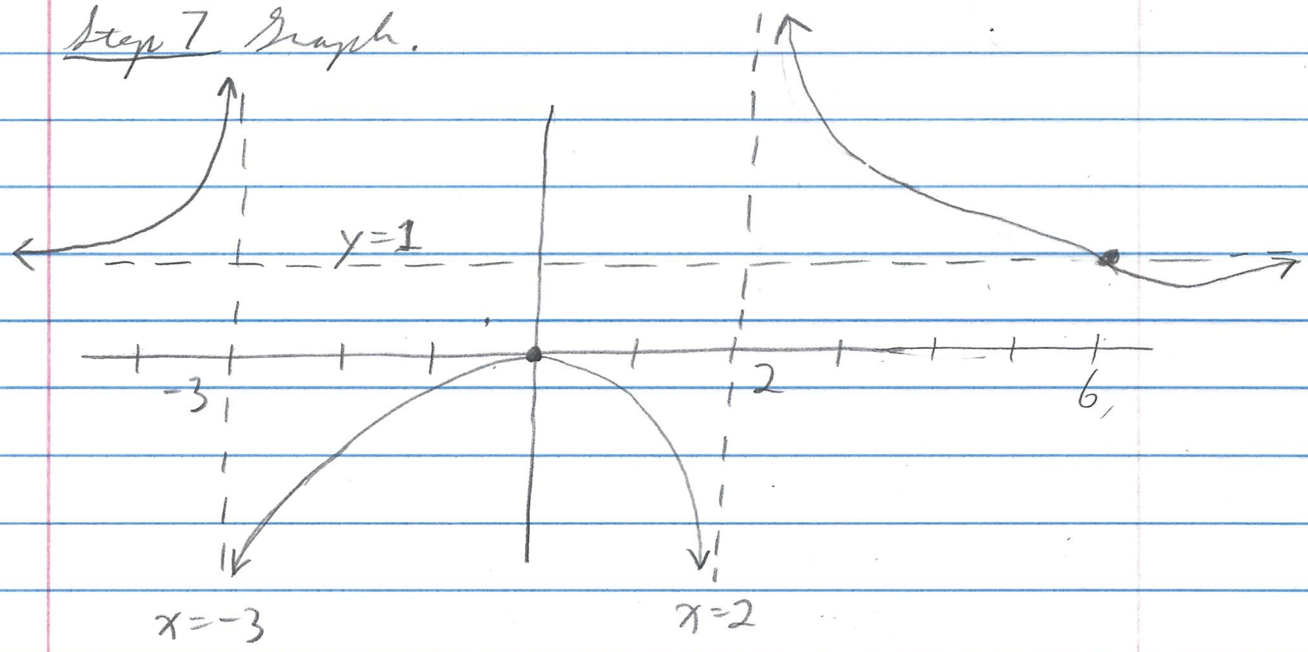
Step 7 Graph.



To conclude Step 3, the graph touches the x-axis at x-intercept $x=0$. □