<u>Exercise 4.4.37</u> Consider $R(x) = \frac{x^2+5x+6}{x+3}$.
Graph R by following Steps 1 through 7.
<u>Solution</u>
<u>Step 1</u> Factor the numerator and denominator and find the domain. We have

$$R(x) = \frac{x^2+5x+6}{x+3} = \frac{(x+3)(x+2)}{(x+3)}.$$

The domain excludes -3 so the $\boxed{\text{domain is } (-\infty,-3) \cup (-3,\infty).}$

<u>Step 2</u> Put the rational function in lowest terms. Since the numerator and denominator have the common factor of $x+3$, so R in lowest terms is $\boxed{R(x) = x+2, \text{ where } x \neq -3.}$

<u>Step 3</u> Locate the intercepts and describe the behavior at each x-intercept. For the y-intercept set $x=0$ and we get

$$R(0) = \frac{(0)^2+5(0)+6}{(0)+3} = \frac{6}{3} = 2,$$ so

$\boxed{\text{the y-intercept is } 2.}$ For x-intercepts we set $y = R(x) = 0$ and consider $x+2=0$ and $x \neq -3$ or $x = -2$. So $\boxed{\text{the x-intercept is } x = -2.}$

<u>Step 4</u> Find the vertical asymptotes. We consider R in lowest terms and see that the vertical asymptotes occur when the denominator is 0. But <u>in lowest terms</u>,

$R(x) = x+2$ where $x \neq -3$ and the denominator is $1 \neq 0$. $\boxed{\text{So there are no vertical asymptotes.}}$

<u>Step 5</u> Locate the horizontal and oblique asymptotes and find points, if any, where the graph intersects them. In lowest terms $R(x) = x+2$ where $x \neq -3$ so the degree of the numerator is $n=1$ and the degree of the denominator is $m=0$. Therefore by Note 4.2.B(3) there is an $\boxed{\text{oblique asymptote of } y = x+2.}$

By Note 4.2.B(2), there is $\boxed{\text{no horizontal asymptote.}}$

<u>Note</u> Since $R(x)$ is the line $x-2$ for $x \neq -3$ (where R is not defined), it seems odd to consider $y = x+2$ as an asymptote. But since R is a rational function, Note 4.2.B does imply that $y = x+2$ is an oblique asymptote.

$\boxed{\text{The graph of } R(x) \text{ intersects the oblique asymptote for all } x \text{ values except } x=-3.}$

<u>Step 6</u> Determine where the graph is above or below the x-axis. We remove the zeros of the numerator and denominator from $\mathbb{R}$ and test the sign of R on these intervals. We get the intervals $(-\infty,-3)$, $(-3,-2)$, and $(-2,\infty)$.

Continued →

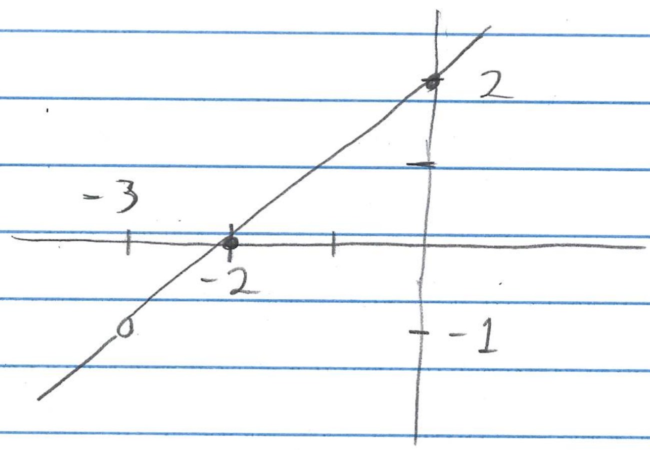
4.4.37 continued 2

Interval	$(-\infty, -3)$	$(-3, -2)$	$(-2, \infty)$
Test Value c	-4	$-5/2$	0
Value of $R(c)$	-2	$-1/2$	2
Conclusion	R negative	R negative	R positive

So R is above the x-axis on $(-2, \infty)$ and R is below the x-axis on $(-\infty, -3) \cup (-3, -2)$.

Step 7 Graph.

Notice that we only need to graph $R(x) = x + 2$ for $x \neq -3$:



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