Exercise 4.5.21 Solve $x^3 - 4x^2 > 0$ algebraically.

Solution

We follow the four steps for solving a polynomial inequality algebraically.

Step 1 Write the inequality with a function on the left hand side, so we set $f(x) = x^3 - 4x^2$.

Step 2 Determine the real zeros of f. We have $f(x) = x^3 - 4x^2 = x^2(x-4) = 0$ and we get the zeros $x = 0$ and $x = 4$.

Step 3 Use the zeros to divide $\mathbb{R}$ into intervals. Removing $x = 0$ and $x = 4$ from $\mathbb{R}$ we get the intervals $(-\infty, 0)$, $(0, 4)$, and $(4, \infty)$.

Step 4 Use a test value c to find the sign on each interval. We have

Interval	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
Test Value c	-1	1	5
$f(c)$	-5	-3	25
Conclusion	f negative	f negative	f positive

So $f(x) = x^3 - 4x^2 > 0$ for $x \in (4, \infty)$. $\qquad \square$