4.5.49

Exercise 4.5.49 Solve algebraically:

$$\frac{x^2(3+x)(x+4)}{(x+5)(x-1)} \geq 0.$$

Solution

We follow the four steps for solving a polynomial or rational inequality algebraically.

Step 1 Write the inequality with a function on the left hand side. So we set

$$f(x) = \frac{x^2(3+x)(x+4)}{(x+5)(x-1)}.$$

Step 2 Determine where f is zero or undefined. f is 0 where the numerator is 0, so the zeros are $x=0$, $x=-3$, and $x=-4$. f is undefined where the denominator is 0, so f is undefined at $x=-5$ and $x=1$.

Step 3 Use the numbers from Step 2 to divide $\mathbb{R}$ into intervals. Using $x=-5$, $x=-4$, $x=-3$, $x=0$, and $x=1$, so the desired intervals are $(-\infty,-5)$, $(-5,-4)$, $(-4,-3)$, $(-3,0)$, $(0,1)$, and $(1,\infty)$.

Step 4 Use a test value c to find the sign in each interval. We have

Interval	$(-\infty,-5)$	$(-5,-4)$	$(-4,-3)$
Test value c	-6	$-9/2$	$-7/2$
$f(c)$	$\frac{(-6)^2(-3)(-2)}{(-1)(-7)}$	$\frac{(\frac{-9}{2})^2(\frac{-3}{2})(\frac{-1}{2})}{(\frac{1}{2})(\frac{-11}{2})}$	$\frac{(\frac{-7}{2})^2(\frac{-1}{2})(\frac{1}{2})}{(\frac{3}{2})(\frac{-9}{2})}$
Conclusion	f positive	f negative	f positive

4.5.49 continued

Interval	$(-3, 0)$	$(0, 1)$	$(1, \infty)$
Test Value c	-1	$1/2$	2
$f(c)$	$\frac{(-1)^2(2)(3)}{(4)(-2)}$	$\frac{(\frac{1}{2})^2(\frac{7}{2})(\frac{9}{2})}{(\frac{11}{2})(\frac{-1}{2})}$	$\frac{(2)^2(5)(6)}{(7)(1)}$
Conclusion	f is negative	f negative	f positive

Since $f(x) = 0$ at $x = 0$, $x = -3$, and $x = -4$ then we have $\boxed{f(x) \geq 0 \text{ on } (-\infty, -5) \cup [-4, -3] \cup \{0\} \cup (1, \infty).}$

□