

Exercise 4.5.61 What is the domain of  $f(x) = \sqrt{x^4 - 16}$ ?

Solution

We cannot take square roots of negatives, so we need  $x^4 - 16 \geq 0$ . Let  $g(x) = x^4 - 16$ .

We have  $g(x) = (x^2 - 4)(x^2 + 4) = (x+2)(x-2)(x^2 + 4) = 0$  for  $x = -2$  and  $x = 2$  (notice  $x^2 + 4$  is never 0).

So we consider the intervals  $(-\infty, -2)$ ,  $(-2, 2)$ , and  $(2, \infty)$ . We then have

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test Value $c$	$c = -3$	$c = 0$	$c = 3$
$g(c)$	65	-16	65
Conclusion	$g$ positive	$g$ negative	$g$ positive

Since  $g(x) = 0$  for  $x = -2$  and  $x = 2$ , we have  $g(x) \geq 0$  (and so the domain of  $f$ ) as

$$\boxed{(-\infty, -2] \cup [2, \infty)}.$$

□