

Exercise 4.6.17 Use the Remainder Theorem (Theorem 4.5.B in the notes) to find the remainder when $f(x) = 4x^6 - 64x^4 - x^2 - 15$ is divided by $x - c = x - (-4) = x + 4$.

Then use the Factor Theorem (Theorem 4.5.C in the notes) to determine whether $x - c$ is a factor of $f(x)$.

Solution

By the Remainder Theorem, the remainder when $f(x)$ is divided by $x - c$ is $f(c)$.

Here, $c = -4$ and

$$\begin{aligned} f(-4) &= 4(-4)^6 - 64(-4)^4 - (-4)^2 - 15 \\ &= 16,384 - 16,384 - 16 - 15 = -31. \end{aligned}$$

By the Factor Theorem, $x - c$ is a factor of $f(x)$ if and only if $f(c) = 0$. Since $f(c) = -31 \neq 0$, then $x - c = x + 4$

is not a factor of $f(x)$.

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