4.6.25

Exercise 4.6.25 Determine the maximum number of real zeros that polynomial function $f(x) = -2x^3 + 5x^2 - x - 7$ may have. Then use Descartes' Rule of Signs (Theorem 4.5.E in the notes) to determine how many positive and how many negative real zeros the polynomial function may have. Do not attempt to find the zeros.

Solution

We have that $f(x)$ is a polynomial function of degree 3, so by Theorem 4.5.D (Number of Real Zeros in the notes) $f(x)$ can have at most 3 real zeros.

The coefficients of $f(x)$ are $-2, 5, -1, -7$ and there are 2 sign variations in the coefficients, so f has either 2 or 0 positive zeros by Descartes' Rule of Signs.

Now $f(-x) = -2(-x)^3 + 5(-x)^2 - (-x) - 7$

$= 2x^3 + 5x^2 + x - 7$

has coefficients $2, 5, 1, -7$ and there is 1 sign variation in the coefficients.

So f has 1 negative zero by Descartes' Rule of Signs.

□