Exercise 4.6.49 Use the Rational Zeros Theorem (Theorem 4.5.F in the notes) to find all the real zeros of the polynomial function $f(x) = 2x^3 - 4x^2 - 10x + 20$. Use the zeros to factor f over the real numbers.

Solution

The leading coefficient of f is $a_3 = 2$ and the constant term is $a_0 = 20$. Since the divisors of a_3 are ± 1 and ± 2, and the divisors of a_0 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$, then by the Rational Zeros Theorem the possible rational zeros of f are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 1/2, \pm 5/2$.

We find $f(1) = 8$, $f(-1) = 24$, $f(2) = 0$, $f(-2) = 8$, $f(4) = 44$, $f(-4) = -132$, $f(5) = 120$, $f(-5) = -280$, $f(1/2) = 57/4$, $f(-1/2) = 95/4$, $f(5/2) = 5/4$, $f(-5/2) = -45/4$. So $x = 2$ is a rational zero and, by the Factor Theorem, $x - 2$ is a factor of $f(x)$. So we divide:

$$
\begin{array}{r l}
 & 2x^2 \qquad\quad - 10 \\
x - 2 & \overline{)\,2x^3 - 4x^2 - 10x + 20} \\
 & -(2x^3 - 4x^2) \\
 & \overline{\qquad -(0 \;\; -10x + 20)} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

4.6.49 continued

So $f(x) = (x-2)(2x^2-10) = 2(x-2)(x^2-5)$.
Now can factor: $x^2 - 5 = (x-\sqrt{5})(x+\sqrt{5})$
(notice that $-\sqrt{5}$ and $\sqrt{5}$ are irrational
zeros of f). Therefore, the zeros of
f are $-\sqrt{5}$, 2, $\sqrt{5}$ and f factors as
$f(x) = 2(x-2)(x-\sqrt{5})(x+\sqrt{5})$.

$\square$