4.6.81

Exercise 4.6.81 Use the Intermediate Value Theorem (Theorem 4.5.5 in the notes) to show that the polynomial function $f(x) = 2x^3 + 6x^2 - 8x + 2$ has a real zero in the interval $[-5, -4]$.

Solution

In the notation of the Intermediate Value Theorem, we have $a = -5$ and $b = -4$.

Now $f(a) = f(-5) = 2(-5)^3 + 6(-5)^2 - 8(-5) + 2$

$= -250 + 150 + 40 + 2 = -58$

and $f(b) = f(-4) = 2(-4)^3 + 6(-4)^2 - 8(-4) + 2$

$= -128 + 96 + 32 + 2 = 2.$

Since $f(a) = f(-5) = -58 < 0$ and $f(b) = f(-4) = 2 > 0$ are of opposite sign then by the Intermediate Value Theorem, f has a real zero between $a = -5$ and $b = -4$. That is, f has a real zero in $[a, b] = [-5, -4]$, as claimed. □