5.1.33

Exercise 5.1.33 For $f(x)=\sqrt{x}$ and $g(x)=2x+5$ find the compositions and domains for (a) $f\circ g$ (b) $g\circ f$ (c) $f\circ f$, and (d) $g\circ g$.

Solution

(a) $(f\circ g)(x) = f(g(x)) = f(2x+5) = \boxed{\sqrt{2x+5}}$.

The domain is all x such that $2x+5 \geq 0$ or $x \geq -5/2$. That is, the domain is $[-5/2, \infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g(\sqrt{x}) = \boxed{2\sqrt{x}+5}$.

The domain is all $x \geq 0$. That is, the domain is $[0, \infty)$.

(c) $(f\circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}}$

$= (x^{1/2})^{1/2} = x^{1/4} = \boxed{\sqrt[4]{x}}$.

The domain is all $x \geq 0$. That is, the domain is $[0, \infty)$.

(d) $(g\circ g)(x) = g(g(x)) = g(2x+5)$

$= 2(2x+5)+5 = \boxed{4x+15}$.

The domain is all real numbers, $\mathbb{R} = (-\infty, \infty)$.

□