

5.1.45

Exercise 5.1.45 For $f(x) = ax + b$ and $g(x) = \frac{1}{a}(x - b)$ (where $a \neq 0$), show that $(f \circ g)(x) = (g \circ f)(x) = x$.

Solution

We have

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{a}(x - b)\right)$$

$$= a\left(\frac{1}{a}(x - b)\right) + b = (x - b) + b = x, \text{ and}$$

$$(g \circ f)(x) = g(f(x)) = g(ax + b)$$

$$= \frac{1}{a}((ax + b) - b) = \frac{1}{a}(ax) = x. \quad \square$$