

Exercise 5.1.47 Find functions  $f$  and  $g$  so that  $f \circ g = H$  where  $H(x) = (2x+3)^4$ .

Solution

For  $H(x) = (2x+3)^4$ , we want the "outer" function to be the function that raises its input to the 4th power, and we want the "inner" function to be  $2x+3$  (the inner function is stuff that is raised to the 4th power in  $H$ ).

So we set  $g(x) = 2x+3$  and  $f(x) = x^4$ .

Composing we have

$$(f \circ g)(x) = f(g(x)) = f(2x+3) = (2x+3)^4 = H(x),$$

as desired. So  $f(x) = x^4$  and  $g(x) = 2x+3$ .

□