

Exercise 5.2.37 Consider $f(x) = x^3 - 8$ and $g(x) = \sqrt[3]{x+8}$. Verify that f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and the domain of g .

Solution

$$\begin{aligned} \text{We have } f(g(x)) &= f(\sqrt[3]{x+8}) = (\sqrt[3]{x+8})^3 - 8 \\ &= (x+8) - 8 = x \end{aligned}$$

$$\begin{aligned} \text{and } g(f(x)) &= g(x^3 - 8) = \sqrt[3]{(x^3 - 8) + 8} \\ &= \sqrt[3]{x^3} = x. \end{aligned}$$

Since the domain of f is $\mathbb{R} = (-\infty, \infty)$ and the domain of g is $\mathbb{R} = (-\infty, \infty)$, then the range of g is $\mathbb{R} = (-\infty, \infty)$ and the range of f is $\mathbb{R} = (-\infty, \infty)$ (since the range of f is the domain of g and the domain of g is the range of f). So

no x values must be excluded.

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