

Exercise 5.2.49 The function $f(x) = x^2 + 4$, $x \geq 0$, is one-to-one. (a) Find its inverse f^{-1} and check your answer. (b) Find the domain and range of f and f^{-1} . (c) Graph f , f^{-1} , and $y = x$ on the same coordinate axes.

Solution

(a) We follow the 3 steps of this section.

First, set $y = f(x)$ so that $y = x^2 + 4$, $x \geq 0$. Interchange the variables x and y to obtain $x = y^2 + 4$, $y \geq 0$. Second, we solve for y :

$$y^2 = x - 4, \quad y \geq 0$$

$$\text{or } |y| = \sqrt{x-4}, \quad y \geq 0$$

$$\text{or } y = \sqrt{x-4} \quad (\text{since } |y| = y \text{ because } y \geq 0).$$

Therefore, $f^{-1}(x) = \sqrt{x-4}$. We check:

$$f(f^{-1}(x)) = f(\sqrt{x-4}) = (\sqrt{x-4})^2 + 4$$

$$= (x-4) + 4 \quad \text{for } x \geq 4$$

$$= x,$$

$$\text{and } f^{-1}(f(x)) = f^{-1}(x^2 + 4), \quad x \geq 0$$

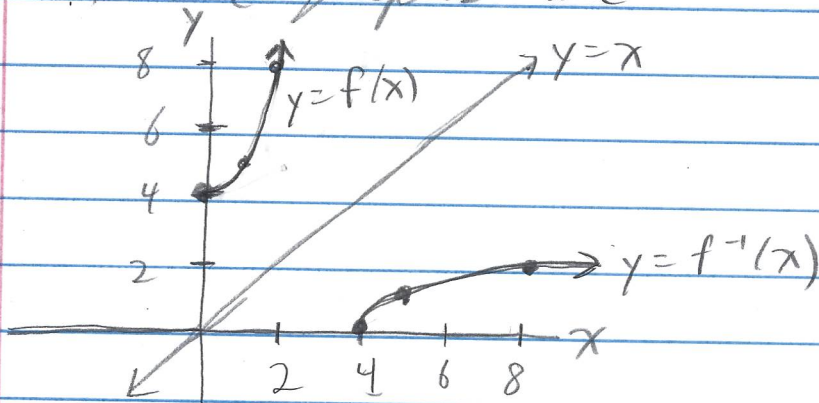
$$= \sqrt{(x^2 + 4) - 4}, \quad x \geq 0$$

$$= \sqrt{x^2} = |x| = x \quad (\text{since } x \geq 0).$$

Third, $f(f^{-1}(x)) = x$ for all $x \geq 4$ (that is, for all x in the domain of f^{-1}) and $f^{-1}(f(x)) = x$ for all $x \geq 0$ (that is, for all x in the domain of f). Therefore, f and f^{-1} are inverses.

(b) The domain of f is $[0, \infty)$ and the domain of f^{-1} is $[4, \infty)$. The domain of one function is the range of the other, so the range of f is $[4, \infty)$ and the range of f^{-1} is $[0, \infty)$.

(c) The graphs are



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