

Exercise 5.3.133 Solve $3^{2x-1} - 4 \cdot 3^x + 9 = 0$.

Solution

We have $3^{2x-1} = 3^{2x} 3^{-1} = \frac{1}{3} 3^{2x} = \frac{1}{3} (3^x)^2$.

So the equation becomes

$$\frac{1}{3} (3^x)^2 - 4(3^x) + 9 = 0, \text{ or}$$

$$\frac{1}{3} ((3^x)^2 - 12(3^x) + 27) = 0, \text{ or}$$

$$\frac{1}{3} ((3^x) - 9)((3^x) - 3) = 0.$$

So the solutions satisfy $3^x - 9 = 0$ and $3^x - 3 = 0$, or $3^x = 3^2$ and $3^x = 3^1$. Since exponential functions are one-to-one then either $x = 2$ or $x = 1$, respectively.

So the solutions are $\boxed{x = 2 \text{ and } x = 1}$.

□