Exercise 5.4.49 Find the domain of $f(x) = \sqrt{\ln x}$.

Solution

We cannot take square roots of negatives, so we need $\ln x \geq 0$. We know from the properties of $\log_a(x)$ that for $a > 1$ the logarithm is increasing and it has an x-intercept of 1. Since $a = e > 1$, then $\log_e(x) = \ln(x)$ is increasing and contains the x-intercept $(1, 0)$. So for all $x \geq 1$ we $\ln(x) \geq 0$ (notice the graph of $y = \ln(x)$). So the domain of the function $f(x) = \sqrt{\ln x}$ is $[1, \infty)$.

$\square$