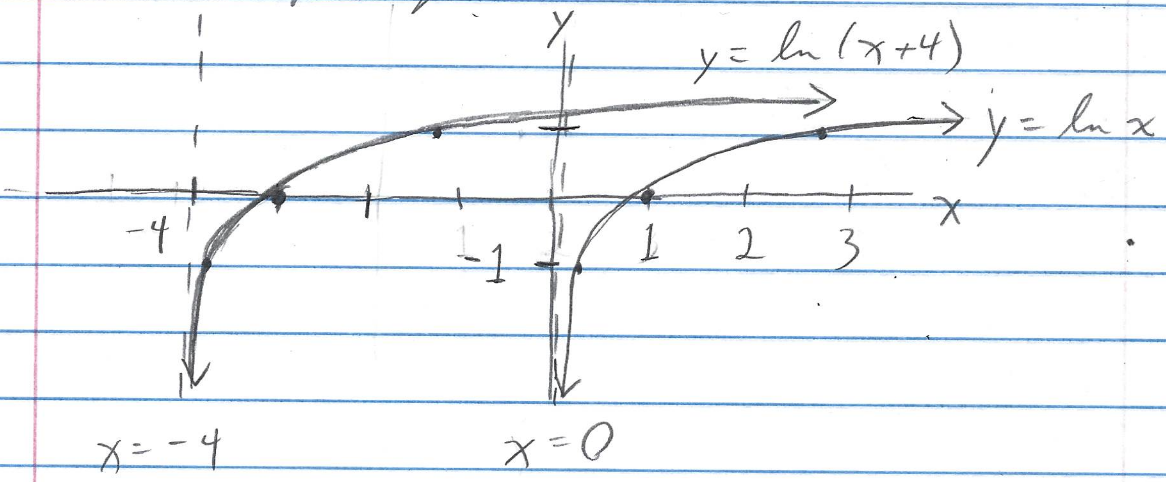
5.4.73

Exercise 5.4.73 Consider $f(x) = \ln(x+4)$.
(a) Find the domain of f. (b) Graph f.
(c) From the graph, determine the range and any asymptotes of f. (d) Find f^{-1}.
(e) Find the domain and range of f^{-1}. (e) Graph f^{-1}.

Solution

(a) We have by the properties of logarithms that the domain of $\log_a(x)$ is $(0,\infty)$.
So the $\boxed{\text{domain of } \log_e(x) = \ln(x) \text{ is } (0,\infty).}$

(b) We get $f(x) = \ln(x+4)$ by replacing x with $x-(-4) = x+4$ in $y = \ln(x)$. So the graph of $y = f(x) = \ln(x+4)$ results from shifting the graph of $y = \ln(x)$ to the left by 4 units:



$\boxed{\text{(c) The range of f is all real numbers and the vertical asymptote is } x=-4.}$

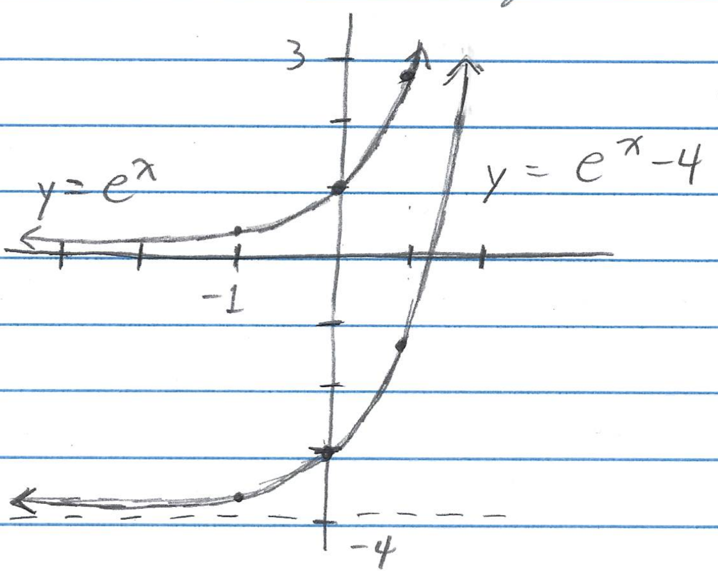
5.4.73 continued

(d) To find f^{-1}, we follow the 3 Steps of Section 5.2. First, set $y = f(x) = \ln(x+4)$ and interchange x and y to get $x = \ln(y+4)$. Second, solve for y. By the definition of logarithm $x = \log_e(y+4) = \ln(y+4)$ means

$$e^x = y+4 \text{ or } \boxed{y = e^x - 4 = f^{-1}(x).}$$

(We omit the third step which is to verify this is the inverse.) The domain of $f^{-1}(x) = e^x - 4$ is $\mathbb{R} = (-\infty, \infty)$ and the range of f^{-1} is the domain of f and so is $(-4, \infty)$.

(e) We can graph $f^{-1}(x) = e^x - 4$ either by shifting the graph of $y = e^x$ down by 4 units or reflecting the graph of $y = f(x) = \ln(x+4)$ about the line $y = x$. In either case we get:



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