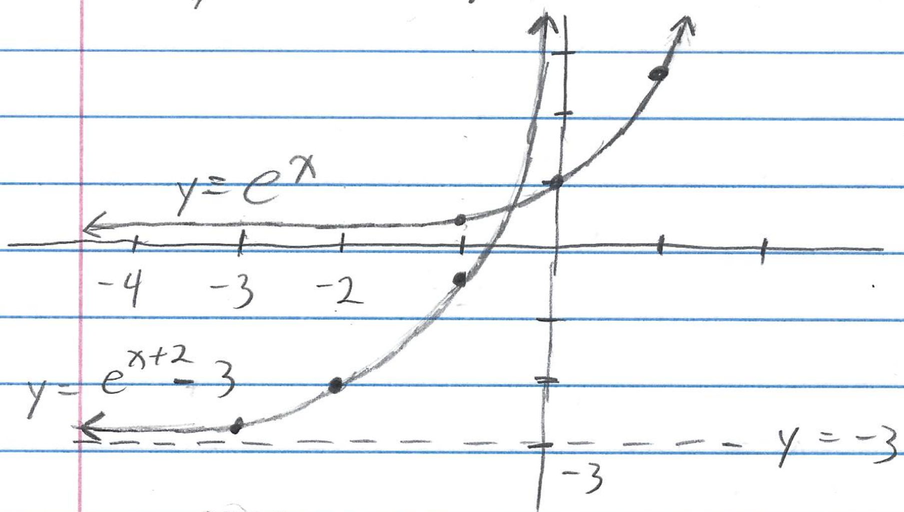
Exercise 5.4.85 Consider $f(x) = e^{x+2} - 3$.
(a) Find the domain of f. (b) Graph f.
(c) From the graph, determine the range and any asymptotes of f. (d) Find f^{-1}.
(e) Find the domain and range of f^{-1}. (f) Graph f^{-1}.

Solution

(a) The domain of all exponential functions are $\mathbb{R} = (-\infty, \infty)$, so $\boxed{\text{the domain of } f \text{ is } \mathbb{R} = (-\infty, \infty)}$.

(b) We start with the graph of $y = e^x$ and replace x with $x - (-2) = x + 2$ to get $y = e^{x+2}$ which is a shift to the left of $y = e^x$ by 2 units. Next, we subtract 3 from $y = e^{x+2}$ to get $f(x) = e^{x+2} - 3$ which is a shift down of $y = e^{x+2}$ by 3 units. So we have:



(c) $\boxed{\text{From the graph, the range is } (-3, \infty) \text{ and } y = -3 \text{ is the horizontal asymptote.}}$

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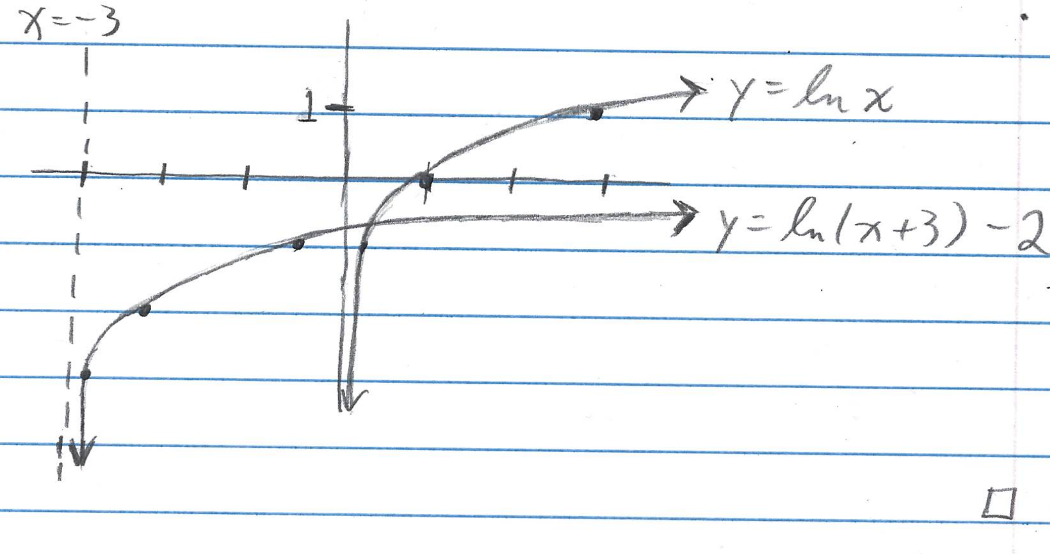
continued

(d) To find f^{-1}, we follow the 3 steps of Section 5.2. First, we set $y = f(x) = e^{x+2} - 3$ and interchange x and y to get $x = e^{y+2} - 3$. Second, we solve for y. We have $x+3 = e^{y+2}$ and by the definition of logarithm this implies that $y+2 = \ln(x+3)$ or $y = \ln(x+3) - 2$. So $\boxed{f^{-1}(x) = \ln(x+3) - 2.}$ (We omit the third step which is to verify this is the inverse.)

(e) We can only take logarithms of positive numbers, so the domain of f^{-1} is $(-3, \infty)$. The range of f^{-1} is the domain of f and so is $\mathbb{R} = (-\infty, \infty)$.

(f) To graph $f^{-1}(x) = \ln(x+3) - 2$, we start with $y = \ln(x)$ and replace x with $x - (-3) = x+3$ (which yields a shift to the left by 3 units) to get $y = \ln(x+3)$. Next we subtract 2 (which yields a shift down by 2 units) to get $f^{-1}(x) = \ln(x+3) - 2$:



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