

Exercise 5.5, 101 Show that

$$\log_a (x + \sqrt{x^2 - 1}) + \log_a (x - \sqrt{x^2 - 1}) = 0.$$

Solution

$$\begin{aligned} & \text{We have } \log_a (x + \sqrt{x^2 - 1}) + \log_a (x - \sqrt{x^2 - 1}) \\ &= \log_a \left( (x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1}) \right) \text{ by Theorem} \\ & \hspace{15em} 5.5, A(3) \\ &= \log_a \left( (x)^2 - (\sqrt{x^2 - 1})^2 \right) = \log_a (x^2 - (x^2 - 1)) \\ &= \log_a (1) = 0 \text{ since } a^0 = 1. \quad \blacksquare \end{aligned}$$