

Exercise 5.5.109 Show that

$$\log_a \left(\frac{M}{N} \right) = \log_a(M) - \log_a(N) \text{ where } a, M, \text{ and } N$$

are positive real numbers and $a \neq 1$.

Solution

$$\text{Let } A = \log_a(M) \text{ and } B = \log_a(N).$$

By the definition of logarithm, these mean

$$a^A = M \text{ and } a^B = N. \text{ Then}$$

$$\log_a(M/N) = \log_a(a^A/a^B) = \log_a(a^{A-B})$$

$$= A - B \text{ by Theorem 5.5.1(2)}$$

$$= \log_a(M) - \log_a(N).$$