

5,5,53

Exercise 5,5,53 Write as a sum and/or difference of logarithms. Express powers as factors:

$$\ln \left(\frac{x^2 - x - 2}{(x+4)^2} \right)^{1/3}, \quad x > 2.$$

Solution

We have

$$\ln \left(\frac{x^2 - x - 2}{(x+4)^2} \right)^{1/3} = \frac{1}{3} \ln \left(\frac{x^2 - x - 2}{(x+4)^2} \right) \quad \text{by Theorem 5.5, A(5)}$$

$$= \frac{1}{3} \left(\ln(x^2 - x - 2) - \ln((x+4)^2) \right) \quad \text{by Theorem 5.5, A(4)}$$

$$= \frac{1}{3} \left(\ln((x-2)(x+1)) - 2 \ln(x+4) \right) \quad \text{by Theorem 5.5, A(5)}$$

$$= \frac{1}{3} \left(\ln(x-2) + \ln(x+1) - 2 \ln(x+4) \right) \quad \text{by Theorem 5.5, A(3)}$$

$$= \boxed{\frac{1}{3} \ln(x-2) + \frac{1}{3} \ln(x+1) - \frac{2}{3} \ln(x+4)}.$$

□