

Exercise 5.6.104 Solve the equation and express irrational solutions in exact form:  $\ln(x^2) = (\ln x)^2$ .

Solution

By Theorem 5.5.A(5),  $\ln(x^2) = 2 \ln x$  so we have  $2 \ln(x) = (\ln(x))^2$  or

$$(\ln(x))^2 - 2 \ln(x) = 0 \text{ or } \ln(x)(\ln(x) - 2) = 0.$$

So we need  $\ln(x) = 0$  or  $\ln(x) = 2$ .

If  $\ln(x) = 0$  then  $e^{\ln(x)} = e^0$  or  $x = 1$ .

If  $\ln(x) = 2$  then  $e^{\ln(x)} = e^2$  or  $x = e^2$ .

So the solutions are  $x = 1$  and  $x = e^2$ .  $\square$