

Exercise 5.6.25 Solve the logarithmic equation and express irrational solutions in exact form:

$$\log_2(x+7) + \log_2(x+8) = 1.$$

Solution

We have $\log_2((x+7)(x+8)) = 1$ by Theorem 5.5.A(3). Exponentiating base 2 gives

$$2^{\log_2((x+7)(x+8))} = 2^1$$

or, by Theorem 5.5.A(2), $(x+7)(x+8) = 2$.

$$\text{or } x^2 + 15x + 56 = 2 \text{ or } x^2 + 15x + 54 = 0.$$

Therefore $(x+6)(x+9) = 0$ so if x is a solution then $x = -6$ or $x = -9$.

Notice that $x = -6$ is in the domain of both $\log_2(x+7)$ and $\log_2(x+8)$ so that $x = -6$ is a solution. But $x = -9$ is in the domain of neither $\log_2(x+7)$ nor $\log_2(x+8)$ since the domain of a logarithm function contains no negative numbers. Therefore, the solution is

$$\boxed{x = -6.}$$

□