

Exercise 5.6.37 Solve the logarithmic equation and express irrational solutions in exact form:

$$2 \log_5 (x-3) - \log_5 (8) = \log_5 (2).$$

Solution

We have

$$\log_5 ((x-3)^2) - \log_5 (8) = \log_5 (2) \quad \text{by Theorem 5.5.A(5)}$$

$$\text{or } \log_5 \frac{(x-3)^2}{8} = \log_5 (2) \quad \text{by Theorem 5.5.A(4)}.$$

Exponentiating base 5 gives

$$5^{\log_5 \frac{(x-3)^2}{8}} = 5^{\log_5 (2)} \quad \text{or, by Theorem}$$

$$5.5.A(1), \quad \frac{(x-3)^2}{8} = 2 \quad \text{or } (x-3)^2 = 16.$$

The possible solutions are $x-3 = -4$ and $x-3 = 4$; that is, $x = -1$ and $x = 7$.

Notice that $x = -1$ is not in the domain of $\log_5 (x-3)$ (since we cannot take a log of a negative number), but $x = 7$ is in the domain of $\log_5 (x-3)$. So the solution is $x = 7$. \square