

Exercise 5.6.65 Solve the exponential equation and express irrational solutions in exact form:

$$16^x + 4^{x+1} - 3 = 0.$$

Solution

We have $(4^2)^x + 4 \cdot 4^x - 3 = 0$ or

$$(4^x)^2 + 4(4^x) - 3 = 0. \text{ Treating this as}$$

a quadratic equation in 4^x , we have by the quadratic formula:

$$4^x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)} = \frac{-4 \pm \sqrt{28}}{2}$$

$$= \frac{-4 \pm \sqrt{4 \cdot 7}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}.$$

But $4^x > 0$ so we cannot have

$$4^x = -2 - \sqrt{7}. \text{ So the solution is } \boxed{x = -2 + \sqrt{7}}.$$

□