5.6.97

Exercise 5.6.97 The resident population of the United States in 2018 was 327 million people and was growing at a rate of 0.7% per year. Assuming that this growth rate continues, the model

$$P(t) = 327(1.007)^{t-2018}$$

represents the population P (in millions of people) in year t.

(a) According to this model, when will the population of the United States be 415 million people?

(b) According to this model, when will the population of the United States be 470 million people?

Solution

(a) The question is $t = ?$ when $P = 415$. So we consider

$$P(t) = 327(1.007)^{t-2018} = 415 \text{ or}$$

$$(1.007)^{t-2018} = \frac{415}{327}.$$ Taking natural logarithms gives $$\ln(1.007)^{t-2018} = \ln\left(\frac{415}{327}\right)$$

or $$(t-2018)\ln(1.007) = \ln\left(\frac{415}{327}\right)$$ by Theorem 5.5.A(5), or

5.6.97 continued

$$t - 2018 = \frac{\ln\left(\frac{415}{327}\right)}{\ln(1.007)} \quad \text{or} \quad t = 2018 + \frac{\ln\left(\frac{415}{327}\right)}{\ln(1.007)}$$

$$\approx 2018 + 34.16 \approx \boxed{2052.}$$

(b) The question is $t = ?$ when $P = 470$. If we consider $P(t) = 327(1.007)^{t-2018} = 470$ or

$(1.007)^{t-2018} = \frac{470}{327}$. Taking natural logarithms gives $\ln(1.007)^{t-2018} = \ln\left(\frac{470}{327}\right)$

or $(t - 2018)\ln(1.007) = \ln\left(\frac{470}{327}\right)$ by Theorem 5.5.A(5), or

$$t - 2018 = \frac{\ln\left(\frac{470}{327}\right)}{\ln(1.007)} \quad \text{or} \quad t = 2018 + \frac{\ln\left(\frac{470}{327}\right)}{\ln(1.007)}$$

$$\approx 2018 + 52 = \boxed{2070.} \qquad \square$$