Exercise 5.8.10 The half-life of radioactive potassium is 1.3 billion years. If 10 grams is present now, how much will be present in 100 years? In 1000 years.

Solution

The amount A of a radioactive material present at time t is $A(t) = A_0 e^{kt}$ where A_0 is the initial amount and $k < 0$. We measure A in grams and t in years. We take "now" as $t = 0$. Since the half-life is 1.3 billion years, then $A = A_0/2$ when $t = 1.3 \times 10^9$ so that

$$\frac{A_0}{2} = A_0 e^{k(1.3 \times 10^9)} \quad \text{or} \quad \frac{1}{2} = e^{k(1.3 \times 10^9)}$$

$$\text{or } \ln\left(\frac{1}{2}\right) = \ln\left(e^{k(1.3 \times 10^9)}\right) = k(1.3 \times 10^9)$$

$$\text{or } k = \frac{\ln(1/2)}{1.3 \times 10^9} = \frac{-\ln(2)}{1.3 \times 10^9}.$$

So, since $A_0 = 10$ grams, the model is

$$A(t) = 10 \exp\left(\left(\frac{-\ln(2)}{1.3 \times 10^9}\right) t\right)$$

(here we represent e^{kt} as $\exp(kt)$, a standard notation for the natural exponential function).

5.8.10 continued

So the amount present in 100 years (when $t = 100$) is

$$A(100) = 10 \exp\left(\left(\frac{-\ln(2)}{1.3 \times 10^9}\right)(100)\right)$$

$$= 10 \exp\left(\frac{-100 \ln(2)}{1.3 \times 10^9}\right) \approx 9.9999995 \text{ grams.}$$

The amount present in 1000 years is

$$A(1000) = 10 \exp\left(\left(\frac{-\ln(2)}{1.3 \times 10^9}\right)(1000)\right)$$

$$= 10 \exp\left(\frac{-1000 \ln(2)}{1.3 \times 10^9}\right) \approx 9.999995 \text{ grams. } \square$$

<u>Notice</u> With such a huge half-life, very very little of the original amount has decayed away in 100 or 1000 years.