Exercise 5.8.13 A pizza baked at 450°F is removed from the oven at 5:00 PM and placed in a room that is a constant 70°F. After 5 minutes, the pizza is at 300°F. (a) At what time can you begin eating the pizza if you want its temperature to be 135°F? (b) Determine the time that needs to elapse before the temperature of the pizza is 160°F. (c) What do you notice about the temperature as time passes?

Solution

Newton's Law of Cooling states that the temperature at time t is

$$u(t) = T + (u_0 - T)e^{kt}$$ where t is time

(we measure time in minutes after 5:00 PM here), T is the temperature of the surrounding medium (we take $T = 70$°F, the temperature of the room), and u_0 is the initial temperature (we take $u_0 = 450$°F, the initial temperature of the pizza when it is removed from the oven). So we have

$$u(t) = 70 + (450-70)e^{kt} = 70 + 380e^{kt}$$

where temperature u is in °F.

Since when $t=5$ we have $u=300$,
so $300 = 70 + 380\, e^{k(5)}$ or $230 = 380 e^{5k}$

or $\dfrac{230}{380} = e^{5k}$ or $\ln\left(\dfrac{230}{380}\right) = \ln\left(e^{5k}\right) = 5k$

or $k = \dfrac{1}{5}\ln\left(\dfrac{230}{380}\right)$. The model is then

$$u(t) = 70 + 380\, e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}.$$

(a) The question is $t = ?$ when $u = 135$,
so we consider

$$135 = 70 + 380\, e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}$$

or $$\frac{65}{380} = e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}$$ or

$$\ln\left(\frac{65}{380}\right) = \ln\left(e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}\right) = \frac{1}{5}\ln\left(\frac{230}{380}\right)t$$

or $$t = \frac{5\ln\left(\frac{65}{380}\right)}{\ln\left(\frac{230}{380}\right)} \approx 17.58 \text{ minutes}.$$

That is, the pizza will be at 135° F
at $\boxed{5{:}17.58 \text{ PM}}$.

(b) The question is $t = ?$ when $u = 160$°F,
so we consider

5.8.13
continued 2

$$160 = 70 + 380\, e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}$$ or

$$\frac{90}{380} = e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}$$ or

$$\ln\left(\frac{90}{380}\right) = \ln\left(e^{\frac{1}{5}\ln\left(\frac{230}{380}\right)t}\right) = \frac{1}{5}\ln\left(\frac{230}{380}\right)t$$ or

$$t = \frac{5\ln\left(\frac{90}{380}\right)}{\ln\left(\frac{230}{380}\right)} \approx \boxed{14.34 \text{ minutes.}}$$

(c) We see that as more time passes, the cooler the pizza gets. Of course since $k = \frac{1}{5}\ln\left(\frac{230}{380}\right) < 0$ (since $0 < \frac{230}{380} < 1$), then for t "large," e^{kt} is "small" (that is, close to 0). So when t is large, $u(t) = 70 + 380e^{kt}$ is close to (and slightly more than) 70°F. That is, the temperature asymptotically approaches 70°F.

□