

Exercise 5.8.25(b,c) The logistic model

$$P(n) = \frac{113.3198}{1 + 0.115 e^{0.0912n}}$$

models the probability that, in a room of n people, no two people share the same birthday. (b) In a room of $n = 15$ people, what is the probability that no two share the same birthday? (c) How many people must be in a room before the probability that no two people share the same birthday falls below 10%?

Solution

(b) With $n = 15$, the probability is

$$P(15) = \frac{113.3198}{1 + 0.115 e^{0.0912(15)}} \approx \boxed{78.06\%}$$

(c) We solve for n in the equation $P(n) = 10\%$:

$$P(n) = \frac{113.3198}{1 + 0.115 e^{0.0912n}} = 10$$

$$\text{or } 11.33198 = 1 + 0.115 e^{0.0912n} \quad \text{or}$$

$$e^{0.0912n} = \frac{10.33198}{0.115} \quad \text{or (taking natural}$$

$$\text{logs)} \quad 0.0912n = \ln\left(\frac{10.33198}{0.115}\right) \quad \text{or}$$

$$n = \frac{\ln\left(\frac{10.33198}{0.115}\right)}{0.0912} \approx 49.3.$$

So we need at least
50 people. \square