Exercise 5.8.5 The population of a colony of mosquitoes obeys the law of uninhibited growth. (a) If N is the population of the colony and t is the time in days, express N as a function of t. (b) If there are 1000 mosquitoes initially and there are 1800 after 1 day, what is the size of the colony after 3 days. (c) How long is it until there are 10,000 mosquitoes?

Solution

(a) In the law of uninhibited growth we have $A(t) = A_0 e^{kt}$ or $\boxed{N(t) = N_0 e^{kt}}$.

(b) N_0 is the initial amount, so $N_0 = 1000$. With t in days, we have $N(t) = 1000 e^{kt}$ for some k. If there are 1800 when $t = 1$, then $1800 = 1000 e^{k(1)} = 1000 e^{k}$

or $e^{k} = \frac{1800}{1000} = \frac{9}{5}$ or $\ln(e^{k}) = \ln\left(\frac{9}{5}\right)$

or $k = \ln(9/5)$. So the model is

$N(t) = 1000 e^{t \ln(9/5)}$. The size of the colony when $t = 3$ days is

$N(3) = 1000 e^{(3) \ln(9/5)} = \boxed{5832}$

5.8.5 continued

(c) The question is $t = ?$ when $N = 10{,}000$, so we consider

$$10{,}000 = 1000\, e^{t \ln(9/5)} \quad \text{or} \quad 10 = e^{t \ln(9/5)}.$$

Taking natural logarithms gives

$$\ln(10) = \ln\left(e^{t \ln(9/5)}\right) = t \ln(9/5)$$

by Theorem 5.5.A(2), or

$$t = \frac{\ln(10)}{\ln(9/5)} \approx \boxed{3.92 \text{ days.}}$$

$\square$