5.8.9

Exercise 5.8.9 The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?

Solution

The amount A of a radioactive material present at time t is $A(t) = A_0 e^{kt}$ where A_0 is the initial amount and $k < 0$. We A in grams, t in years, and have $A_0 = 10$. We take "now" at $t = 0$. Since the half-life is 1690 years, then $A = A_0/2 = 5$ when $t = 1690$ so that $5 = 10 e^{k(1690)}$ or

$\frac{1}{2} = e^{1690k}$ or $\ln\left(\frac{1}{2}\right) = \ln\left(e^{1690k}\right) = 1690k$

or $k = \frac{\ln(1/2)}{1690} = \frac{-\ln(2)}{1690}$. Therefore

$A(t) = 10 e^{\frac{-\ln(2)}{1690} t}$ and when $t = 50$ years the amount present is

$A(50) = 10 e^{\frac{-\ln(2)}{1690}(50)} \approx \boxed{9.797 \text{ grams}}$ □