Exercise A.3.129 Determine the number that should be added to $x^2 - \frac{1}{2}x$ to complete the square. Then factor the new expression.

Solution

We have $x^2 - \frac{1}{2}x$ is of the form $x^2 - bx$ where $b = 1/2$. So we add $(b/2)^2 = \left(\frac{1/2}{2}\right)^2 = \frac{1}{16}$ and from the formula

$$x^2 - bx + \left(\frac{b}{2}\right)^2 = \left(x - \frac{b}{2}\right)^2$$

we have

$$\boxed{x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{(1/2)}{2}\right)^2 = \left(x - \frac{1}{4}\right)^2.} \quad \square$$