A.3.69

Exercise A.3.69 Find the quotient and the remainder when $2x^4 - 3x^3 + x + 1$ is divided by $2x^2 + x + 1$. Check your work by verifying that

(Quotient)(Divisor) + Remainder = Dividend.

Solution

We perform long division:

$$
\begin{array}{r}
x^2 - 2x + 1/2 \\
2x^2 + x + 1 \overline{\big)\, 2x^4 - 3x^3 \quad\quad + x + 1} \\
-(2x^4 + x^3 + x^2) \\
\hline
-4x^3 - x^2 + x \\
-(-4x^3 - 2x^2 - 2x) \\
\hline
x^2 + 3x + 1 \\
-(x^2 + \frac{1}{2}x + \frac{1}{2}) \\
\hline
\frac{5}{2}x + \frac{1}{2}.
\end{array}
$$

So $\boxed{\text{the quotient is } x^2 - 2x + 1/2 \text{ and the remainder is } \frac{5}{2}x + \frac{1}{2}.}$

Checking, we have (Quotient)(Divisor) + (Remainder) =

$$\left(x^2 - 2x + \frac{1}{2}\right)(2x^2 + x + 1) + \frac{5}{2}x + \frac{1}{2}$$

$$= x^2(2x^2 + x + 1) - 2x(2x^2 + x + 1)$$

$$+ \frac{1}{2}(2x^2 + x + 1) + \frac{5}{2}x + \frac{1}{2}$$

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continued

$$= 2x^4 + x^3 + x^2 - 4x^3 - 2x^2 - 2x$$
$$+ x^2 + \frac{1}{2}x + \frac{1}{2} + \frac{5}{2}x + \frac{1}{2}$$

$$= 2x^4 - 3x^3 + x + 1 = (\text{Dividend}),$$

verifying that our answer is correct. □